The Deficiency of a Solid Angle

If you're given a 3D solid angle (i.e., a corner of a polyhedron or something) then you can try to measure how "pointy" the angle is by using the **plane angles**.

Definition: Given a solid angle with plane angles $A_1, A_2, ...A_n$, we define the **deficiency of the solid angle** to be the "gap" left over if we were to unfold the solid angle and lay it flat. In other words,

$$\text{deficiency of the solid angle } = 2\pi - (A_1 + A_2 + \cdots + A_n).$$

(1) Compute the deficiency (in radians) of a corner of...

(a) a cube.

(b) a tetrahedron.

(c) an octahedron.

(d) an icosahedron.

(e) a dodecahedron.

(2) OK. Now what do you get when you add up the deficiencies of ALL the vertices of

(a) the cube:

(b) the tetrahedron:

(c) the octahedron:

(d) the icosahedron:

(e) the dodecahedron:

Make a conjecture about the sum of the deficiencies of a general polyhedron: