MA 323 **1/24/02**

Combinatorial Geometry

First set of probelms!

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(1) OK. Now you've seen Euler's Formula : Given a connected planar graph with V vertices, E edges, and F faces, we have $V - E + F = 2$.

Use Euler's Formula to prove that there are only 5 Platonic solids, the tetrahedron, cube, octahedron, icosahedron, and dodecahedron. (Remember, a Platonic solid is a polyhedron (planar graph) where all vertices have the same degree and each face has the same number of sides. There are hints for this problem on the web page, in the Planar Graph Theory notes.)

(2) There are three houses on Avenue Y, and on Avenue P there are three service companies, the electric, water, and gas companies. Each company needs to run an uninterrupted pipe to each of the three houses. Due to the fact that Avenue Y and Avenue P exist in 2 dimensional space, no two pipes can cross. Can this be done? If so, draw a picture to show how. If not, prove it.

(3) Use Euler's Formula to prove that every planar graph must contain a vertex of degree 5 or less.

(4) A planar graph G is **self-dual** is $G = G^*$. We saw that the tetrahedron is self-dual. Find more. Can you find a whole family of polyhedra that are self-dual?

(5) I have a *mystery polyhedron*, P, and I tell you that P has **only** triangle faces. I also tell you that P has exactly 8 vertices. How many faces must P have? Draw a graph of what this polyhedron might look like.

(6) Like problem (5), suppose we have that the mystery polyhedron P has only triangle faces, but we don't know how many vertices it has. So let V be the number of vertices.

(a) Find a formula that relates the number of faces F to the number of edges E. (Hint: count the number of edges around each face.)

(b) Then plug this into Euler's Formula to get a formula that relates V and F. (If you can do this, it will make doing number (5) a LOT easier!)

(7) Come up with polyhedra made from Sonobé units that requires 4 colors to properly color.