MA 323A **3/5/02**

Combinatorial Geometry
Problem Set Five to keep you alive!
do any three

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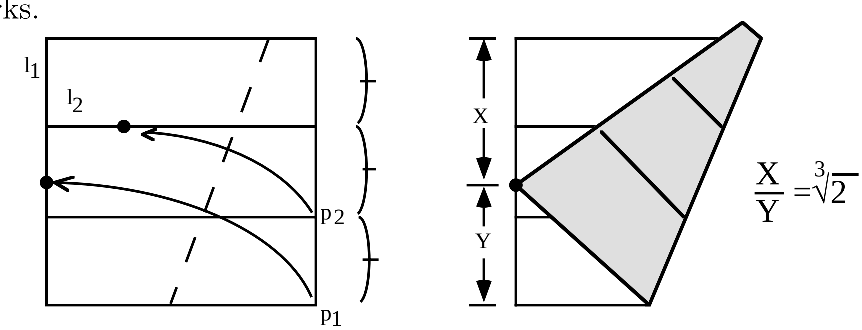
(1) (left over from last time) Prove that the "fold a point to the circle over and over again" trick that we saw really generates an ellipse.

(2) (left over from last time, if you haven't done it yet!) How would you use Axiom 5 to "fold" a hyperbola?

(3) (Hard) Prove that Axiom 6 really does solve a cubic equation. I suggest doing the following: Let the line L_1 be $y = -1$, the point $p_1 = (0, 1)$, and let the point p_2 be denoted by (a, b). (If you want, take $a = 1, b = 1$ or something like that. Or leave it arbitrary.) Then you want to see what the image p_2' of the point p_2 is when we fold p_1 onto L_1. Let this image point $p_2' = (x, y)$ and try to find an equation involving just x, y, a, and b. Hopefully it'll be cubic in terms of x and/or y!

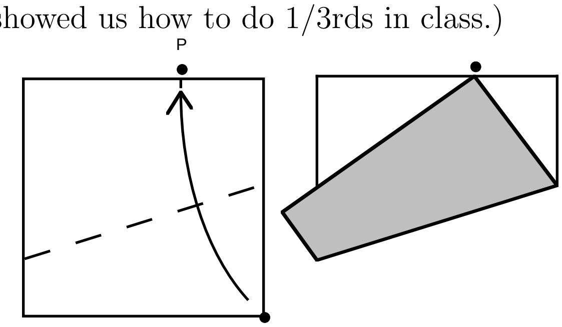
(4) Below is shown how you can use axiom 6 to construct the cube root of 2. Prove that it works.



(5) Find a method of creating arbitraty square roots by folding. (Say, a line segment of length $\sqrt{x}$ for any x.)

(6) We saw in class how to divide the side of a square into 1/3 (or equivalently, 2/3) by considering the crease lines $y = x$ and $y = -2(x - 1)$, where the lower left corner of the square is $(0, 0)$. Modify this method to find a way to divide the side of the square into 1/5ths. Can you generalize to $1/n$ths?

(7) Take a square piece of paper and pick a random point P along the top edge. Then fold the bottom right corner to the point P. (See below.)
(a) What can you prove about the triangles you see? (This is called Haga's Theorem.)
(b) Use part (a) to come up with an exact way to fold the side of a square into 1/5ths. (Since Chris already showed us how to do 1/3rds in class.)



(8) Make a torus out of PHiZZ units! I suggest one of the versions in the 81-105 unit range in the “Notes on Making Modular Tori” section of the course web page. 3-edge color it, if you can!

(9) Find a way to imbedd K_7 (the complete graph on seven vertices) on the torus, using the “fundamental domain” method for drawing a torus on paper.