Name:

MA 323A **4/28/05**

Combinatorial Geometry
The Take Home Final Exam!

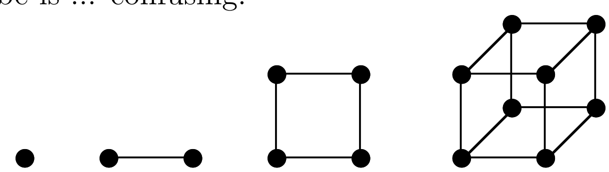
This is an *open notes* exam, without time limit except that it must be turned in by **11:30 a.m. Wednesday, May 11th, 2005**. No collaboration is permitted. You may use the class web page and class notes freely without attribution. You may **NOT** use other web pages for help. If you have a question about the interpretation of these instructions, please ask me.

In order to receive credit on the exam, you must sign below to indicate that you did not collaborate, give help, or receive help from any living sources other than the instructor.

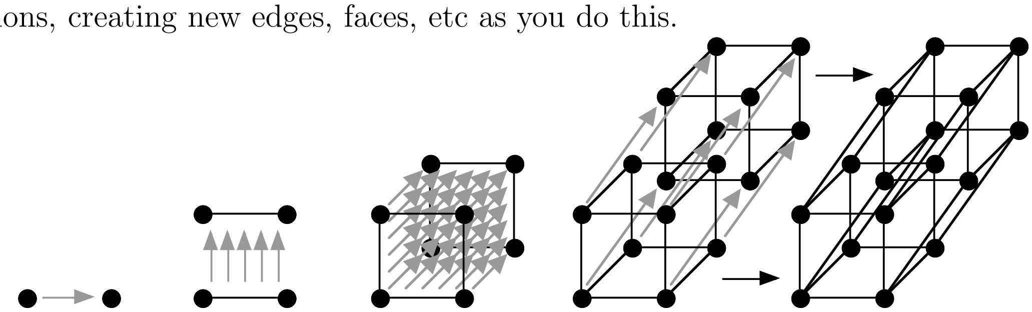
Your signature:

Instructions: This exam contains five problems.

(1) **Hypercubes:** A zero dimensional "cube" is really simple. It's just a single point.
A 1-dimensional cube is just a line segment (with two endpoints).
A 2-dimensional cube is, well, a square.
A 3-dimensional cube is what you're familiar with.
A 4-dimensional cube is ... confusing.



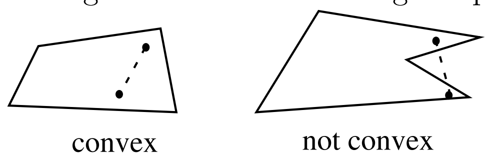
You can go from an n-dimensional cube to the next dimension $(n+1)$ by making another copy of the n-dimensional cube and "mooshing" it along the new axis of your $(n+1)$ dimensions, creating new edges, faces, etc as you do this.



Answer the following questions with proof:
(a) How many vertices does an n-dimensional cube have?
(b) How many edges does an n-dimensional cube have?
(c) How many faces does an n-dimensional cube have?
BONUS: Find (and try to prove it) a formula for the number of k-dimensional faces in an n-dimensional cube.

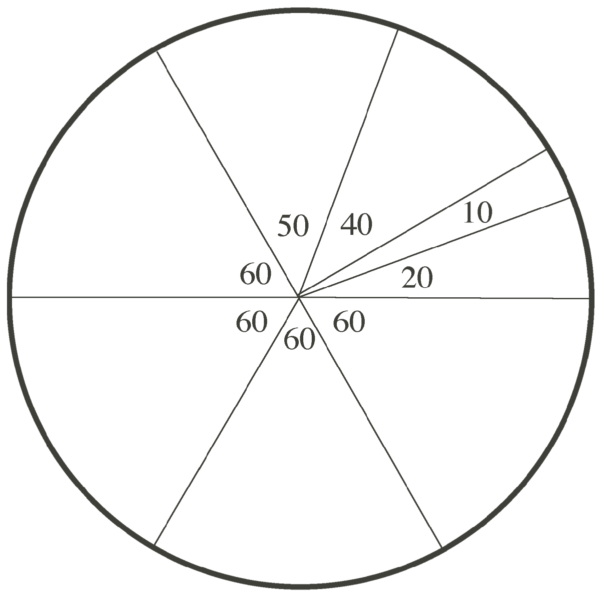
(2) Is it possible for a polyhedron have exactly 7 edges? Why or why not?

(3) A polygon is called **convex** if you can travel from any point in the polygon to any other point in the polygon along a straight line without leaving the polygon. (See below.)

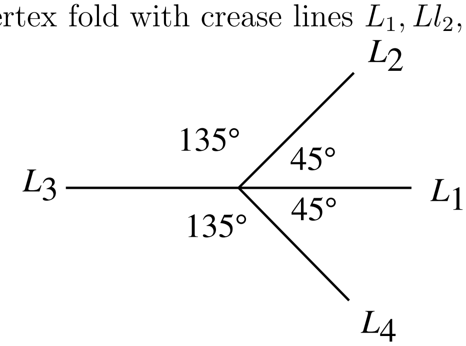


Prove that if F is a face (region) in the crease pattern for a flat origami model, then F must be a convex polygon.

(4) How many valid ways are there to assign mountain and valley creases to the flat vertex fold shown below?



(5) Below is shown a flat vertex fold with crease lines L_1, Ll_2, L_3, L_4.



Assume that this vertex is at the origin in the plane. Show that

$$R(L_1) \cdot R(L_2) \cdot R(L_3) \cdot R(L_4) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Where each $R(L_i)$ is the matrix that reflects the plane about the line L_i, and the $\cdot$ is matrix multiplication.