MA 323 **1/26/05**

Combinatorial Geometry

First set of probelms!

Instructions: Do **two** of the below problems. You may work together, but tell me whom you work with. (And everyone has to write up their own solutions using their own words.) These are WRITE-UPS, so you must EXPLAIN what you're doing and give PROOFS. They should read like something you'd see in a textbook. Any other math person should be able to read your write-up and understand what you're doing without having to read your mind!

Due date: Thursday, February 3, beginning of class.

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(1) OK. Now you've seen Euler's Formula : Given a connected planar graph with V vertices, E edges, and F faces, we have $V - E + F = 2$.

Use Euler's Formula to prove that there are only 5 Platonic Solids. (Hint: Break it down into cases – what can happen if we have only triangle faces? Only square faces? Only pentagon faces? Then argue why we don't need to consider only hexagon or higher faces.)

(2) There are three houses on Avenue Y, and on Avenue P there are three service companies, the electric, water, and gas companies. Each company needs to run an uninterrupted pipe to each of the three houses. Due to the fact that Avenue Y and Avenue P exist in 2 dimensional space, no two pipes can cross. Can this be done? Prove it.

(3) Use Euler's Formula to prove that every planar graph must contain a vertex of degree 5 or less.

(4) A planar graph G is **self-dual** if $G = G^*$. We saw in class that the tetrahedron is self-dual. Find more examples of self-dual polyhedra. In fact, find a whole infinite family of polyhedra that are self-dual.

(5) Construct a PHiZZ unit soccer ball (truncated icosahedron) with a proper 3-edge coloring. Your write-up for this would be to explain in words your coloring algorithm (like finding a Hamilton circuit) and show me the model by bringing it to class.