MA 323 **2/3/05**

Combinatorial Geometry
Trippy Problem Set Number Two

Instructions: Again, do any two of these problems. They're due in a week (Feb 10).

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(1) Is it possible for a Buckyball to have only one hexagon face? Two hexagon faces? Gimme proofs.

(2) The Four Color Theorem is very hard to prove, but the Five Color Theorem isn't so bad. Try to prove that you can color the vertices of any planar graph using five colors. (Feel free to ask for hints! Here's one: start with the fact that any planar graph must have a vertex of degree 5 or less.)

(3) Find formulas for the number of vertices, edges, and faces of a (p, q) spherical Buckyball. Make sure you explain how you got these formulas.

(4) Prove that every 2-connected, cubic planar graph is 3-edge colorable. Use the following **hint:** First use the Four Color Theorem to color the faces of your cubic planar graph. Then somehow use the face coloring to give you a coloring of the edges.

(5) We know now that the soccer ball has a Hamilton circuit. What about the other Buckyballs? Can you use out Buckyball classification method (the (p, q) tile stuff) to find Hamilton circuits on other Buckyballs?

(6) Is the below planar graph a polyhedron? Why or why not?