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On relation between mountain-creases and valley-creases of a flat origami

By

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§ 0. Notation.

Let R be the real euclidian line, R^2 be the real euclidian plane. Let $\text{Isom}(R)$ be the group of isometric automorphisms. For a segment or a half line Q , we denote by $R(Q) \in \text{Isom}(R^2)$ the reflection whose axis is the line including Q . Moreover, we denote by D , the unit disk with center $O(0,0)$.

§ 1. Flat origamis of D

Definition 1.1 (flat origamis of the unit disk D): A locally finite cell decomposition K of D is called a flat origami disk if the following conditions are satisfied,

- (i) For any 1-cell of K , its end point is the center $O(0,0)$.
- (ii) For an arbitrary closed curve γ in D such that γ does not pass through the center O and intersects 1-celles (edges) $Q_1, \dots, Q_r$ of K transversally in this order, the following condition (We call this condition by "flat condition") holds:

$$R(Q_1) \cdots R(Q_r) = \text{identity}.$$

Remark 1.2 A flat origami disk K represents the "development chart" of an origami of the disk which is folded in flat though its valley creases and its mountain creases are not distinguished (Fig.1).

Remark 1.3 For any flat origami disk K , the flat condition is equivalent to the following conditions:

- (i) K has even 1-cells $Q_1, Q_2, Q_3, \dots, Q_{2m}$.
- (ii) Let α_i be the angle between Q_i and Q_{i+1} , α_{2m} be the angle between Q_{2m} and Q_1 , then we have

$$\alpha_1 + \alpha_3 + \alpha_5 + \cdots + \alpha_{2m-1} = \alpha_2 + \alpha_4 + \alpha_6 + \cdots + \alpha_{2m} = 2\pi,$$

§ 2. Mountain-creases and Valley-creases

When a sheet of paper is folded in front or folded behind like Fig.1, the crease is respectively called by a valley-crease and a mountain-crease. Furthermore valley-creases and mountain-creases are represented by dashed lines and chain lines.

Definition 2.1 For an origami K folded in flat, we define a map $C: \{1\text{-cells of } K\} \rightarrow \{\pm 1\}$ such that $C(Q)=1$ if Q is a valley-crease and $C(Q)=-1$ if Q is a mountain-crease. Moreover, C is called by a realization of K . In the case that K is a flat origami disk with center p , we call it by a local realization at p .

We let fold flat origamis in Fig.2.1. Can you fold them? Perhaps, you can fold all. Next, you let fold flat origamis as shown in Fig.2.2. Though they are consist of the flat origami as shown in Fig.2.1(1), you can fold only origami(1). It is imposible to fold the origami (2) in flat. Why only the origami (2) is not folded in flat? See Fig.2.3. If you fold this origami, then you find that two faces σ_1, σ_2 meet above the face τ . So we have

Proposition 2.2 For any flat origami disk K , if $\alpha_{i+1} < \alpha_i$ and $\alpha_{i+1} < \alpha_{i+2}$ then $C(Q_i) = -C(Q_{i+1})$.

Collorary 2.3 For a flat origami disk K which has 4 creases as shown in Fig.2.4, if $\alpha_2 < \alpha_1$ and $\alpha_2 < \alpha_3$ then $C(Q_1) = -C(Q_2)$ and $C(Q_3) = C(Q_4)$.

Example 2.4 A flat origami K as shown in Fig.2.5(1) has many symetry. This kind of origamis are usually called by "平織り = HIRAORI" in Japan. "HIRAORI" was found by S. Fujimoto and Y. Momotani independently. "平 = HIRA" means flat. "織り = ORI" means texture. "HIRAORI" is invaliant under 2-dimensional crstallographic group actions. So we call them by crystallographic flat origamis in this paper. They have many realization. We can find a realization of K as following:

- ① Make two kind of trapezoid tiles as shown in Fig.2.5(2).
- ② Arrange them on the paper like Fig.2.5(3).
- ③ Decide the valley-creases and mountain-creases according to the tiling ② (Fig.2.5(4)).

Example 2.5 All realization of flat origami K as shown in Fig.2.6

(1) are obtained by arrangement of the two kind of parallelogram tiles as shown in Fig.2.6(2).

Example 2.6 The flat origami K as shown in Fig.2.7 has no realization. Fig.2.8, Prop.2.2 and Cor.2.3 show you the reason.

§ 3. Can we fold any flat origami disk in flat?

Fig.2.7 is complex, so it is impossible to fold it in flat. Now, for any flat origami disk, can we fold it in flat? The answer is

Theorem 2.4 Any flat origami disk has (local) realization, ie we can always fold a flat origami disk in flat.

Proof See Fig.2.9.

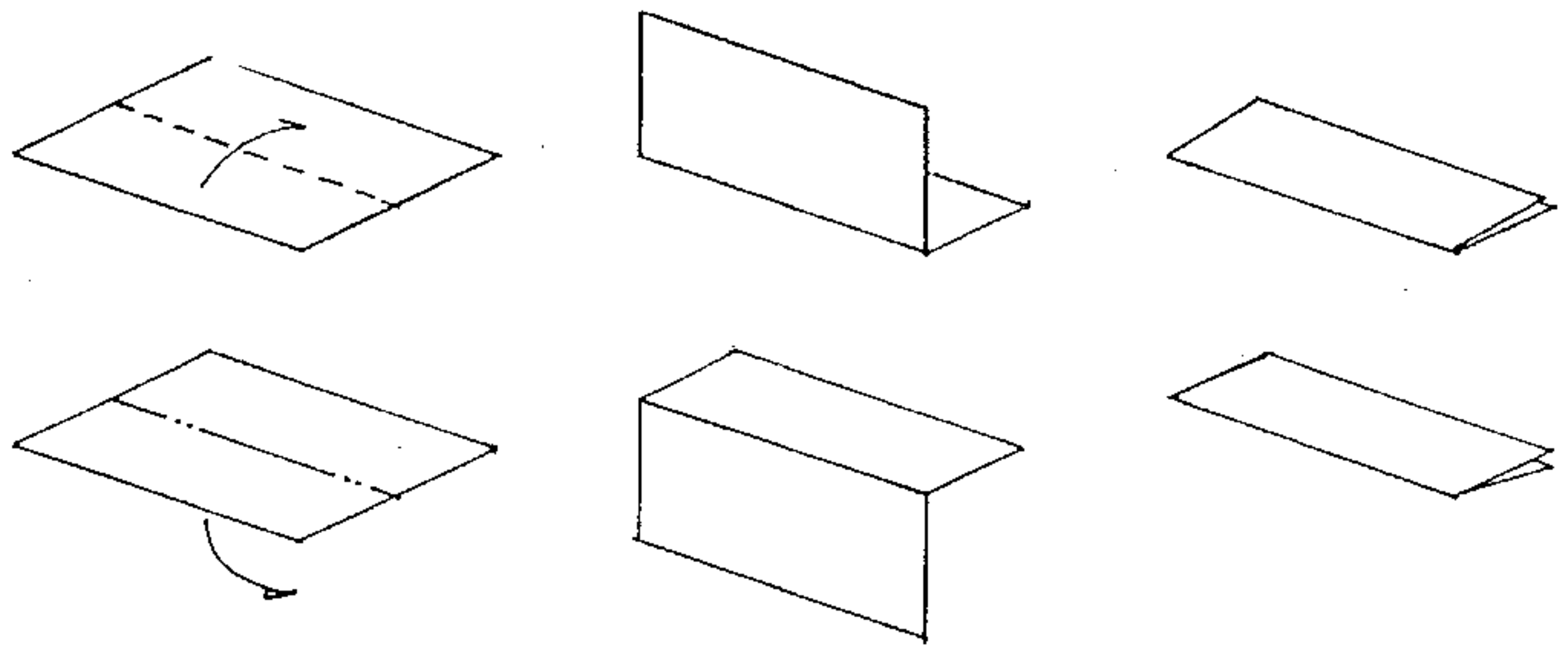


Fig. 1

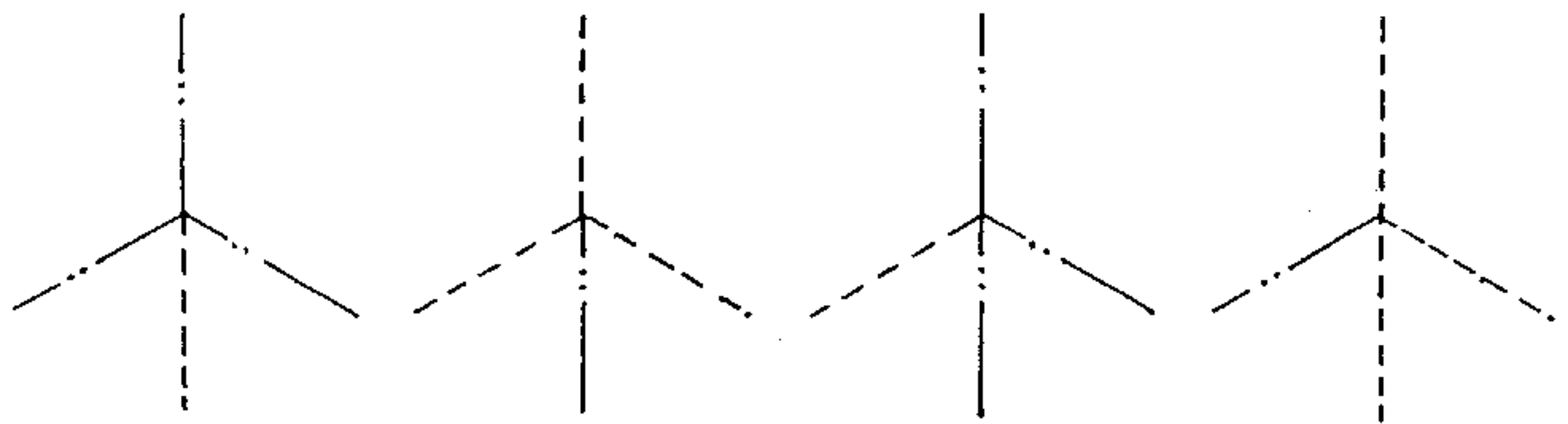
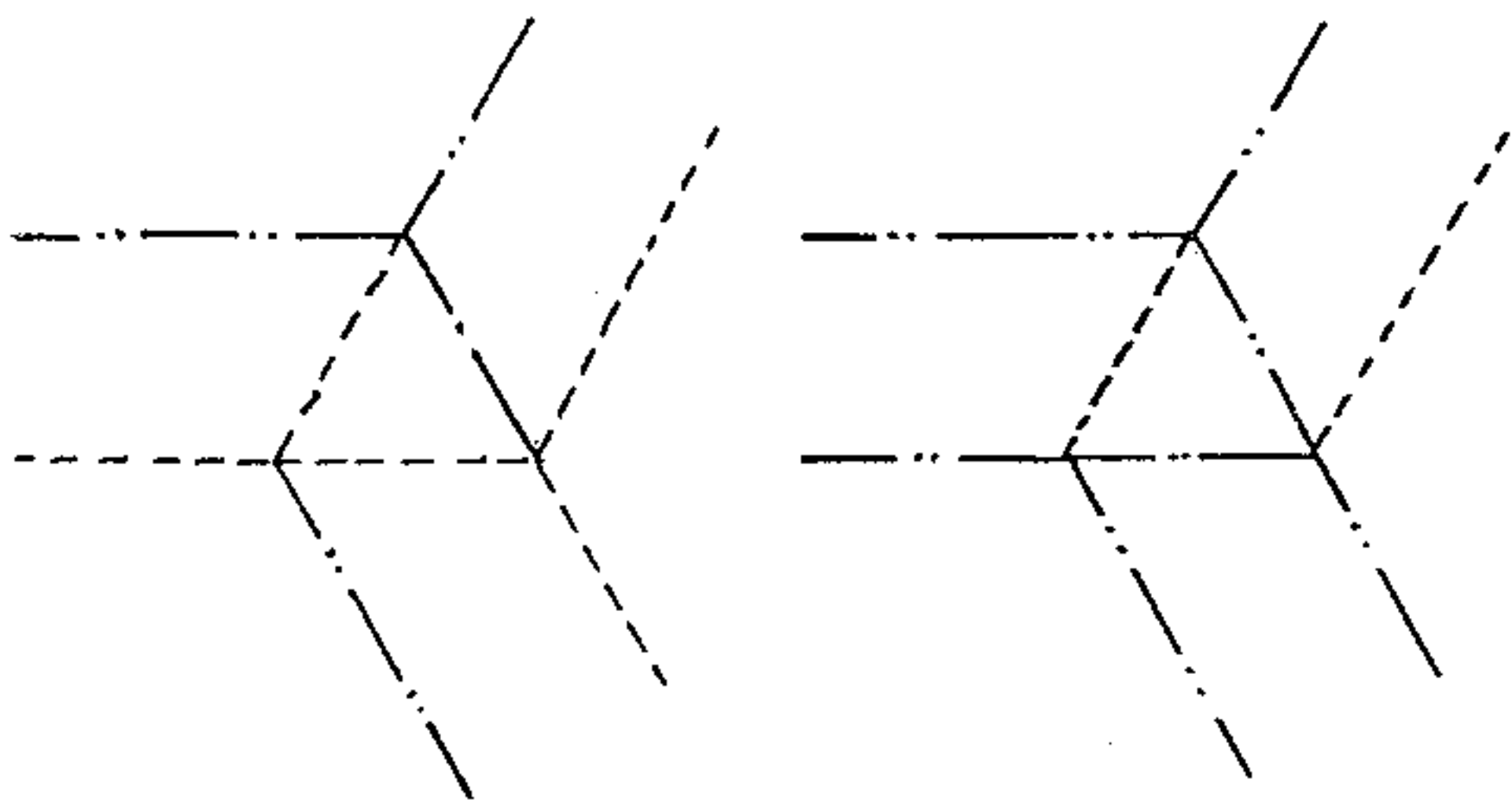


Fig. 2.1



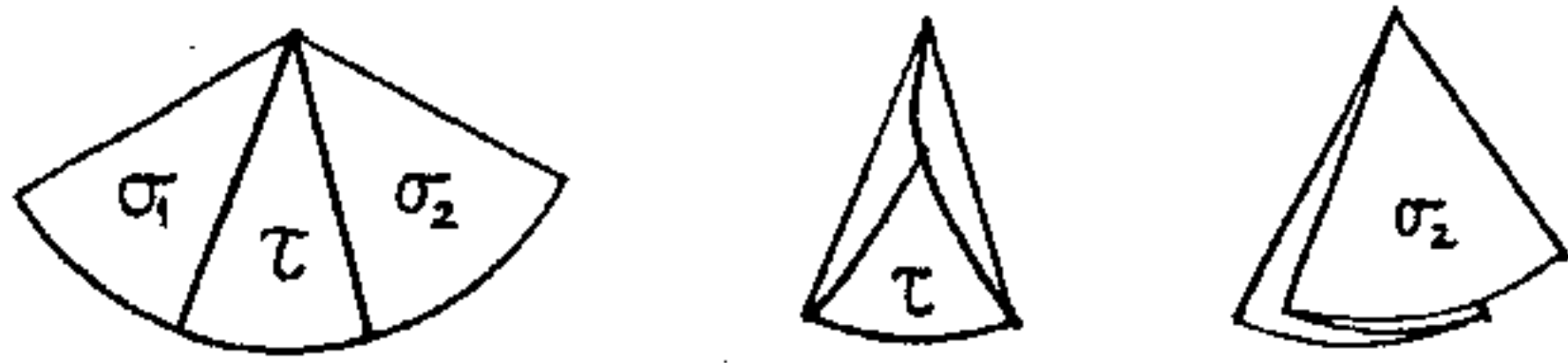


Fig. 2.3

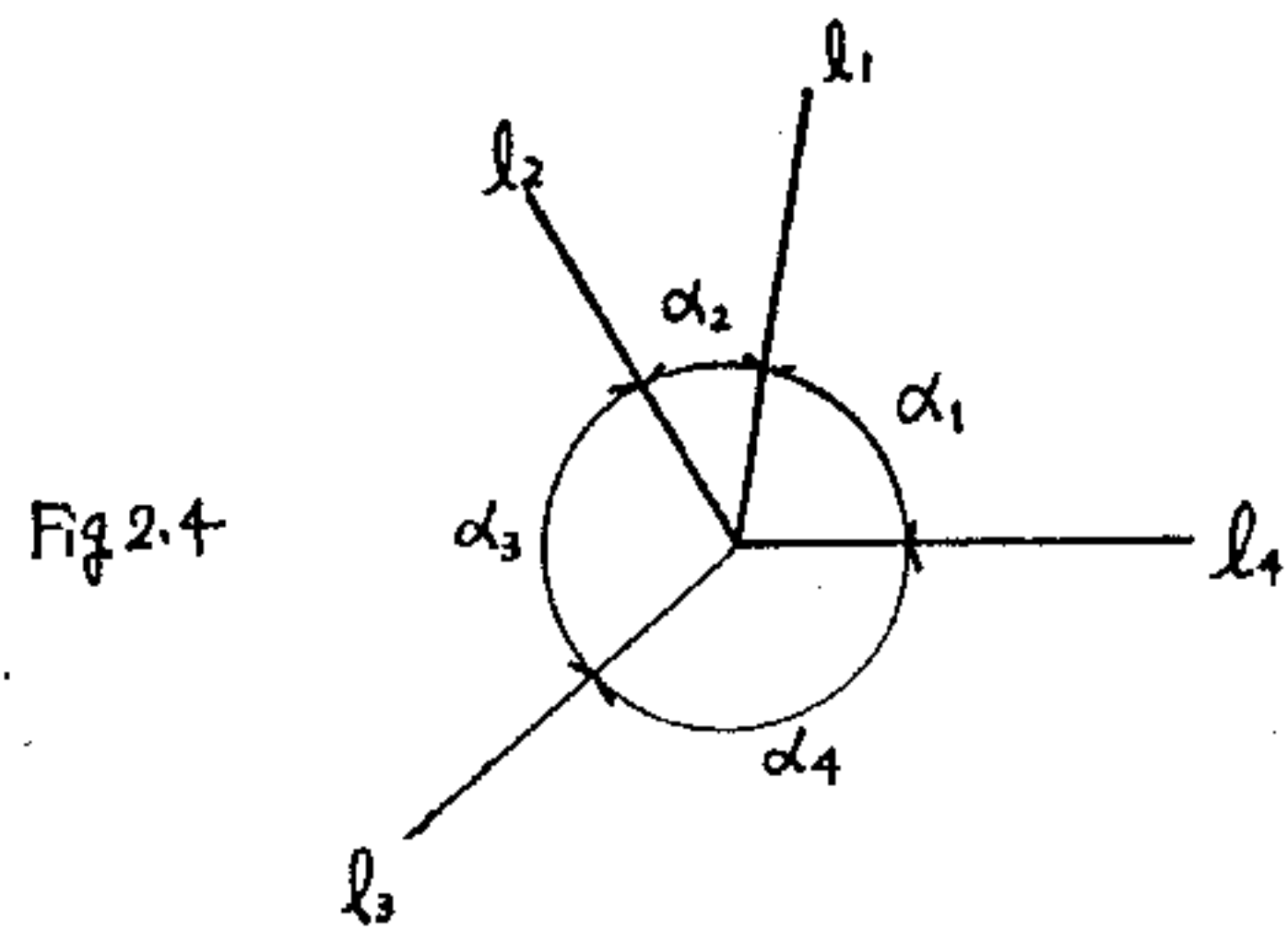
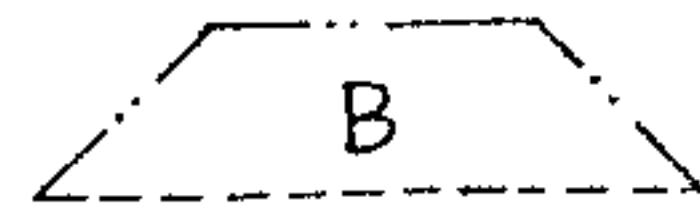
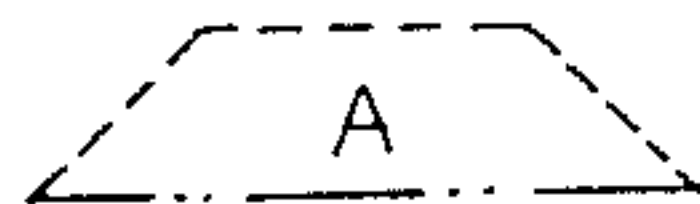
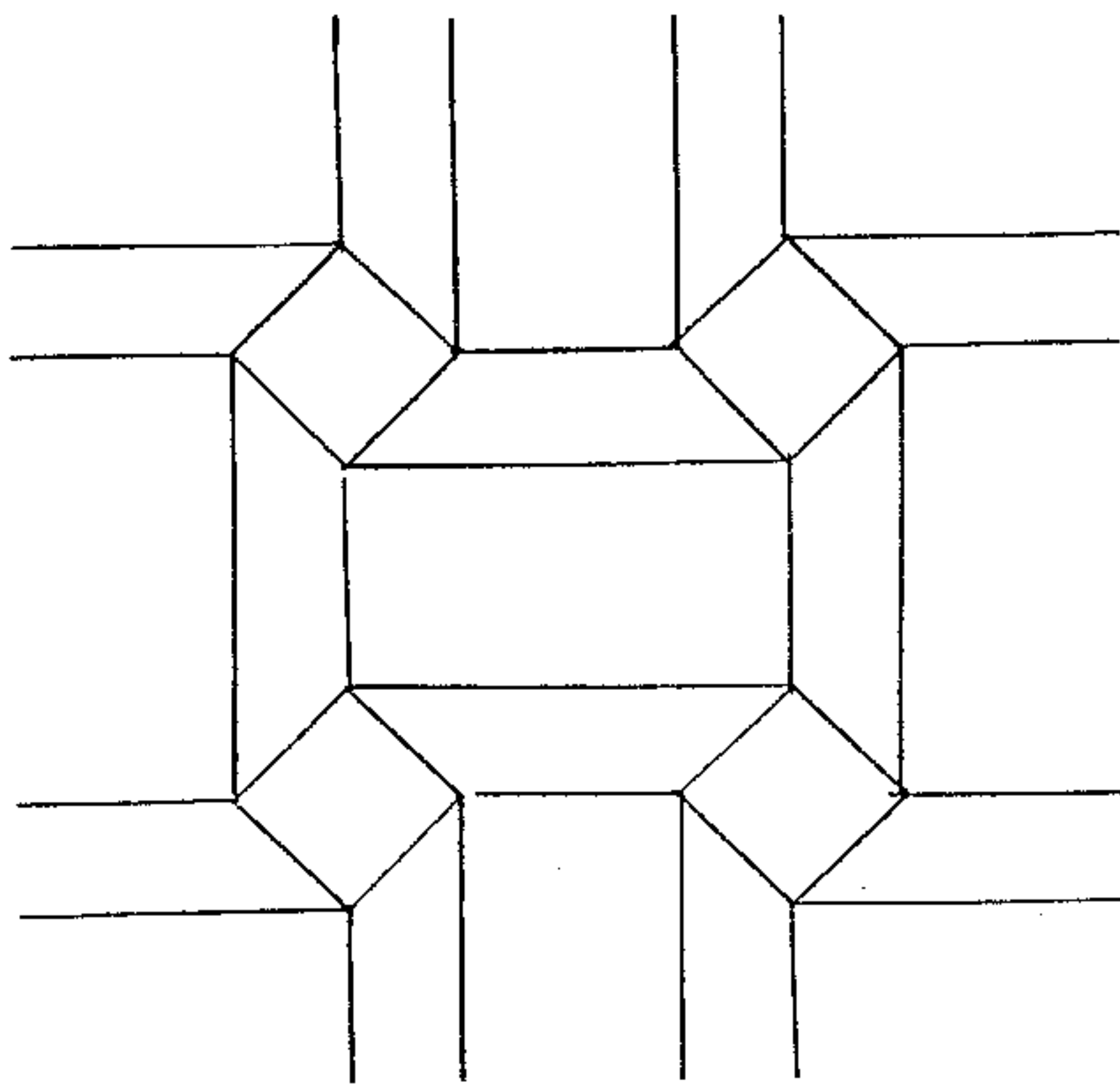
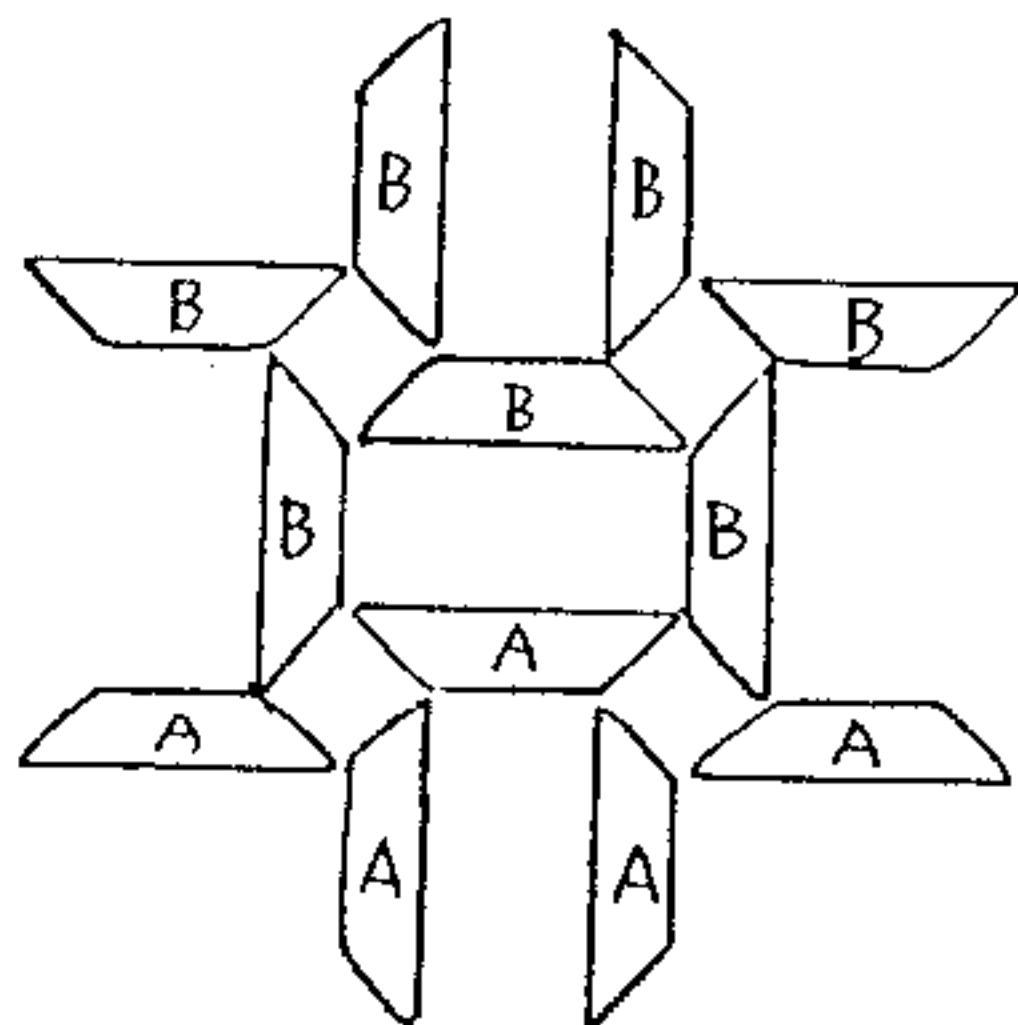
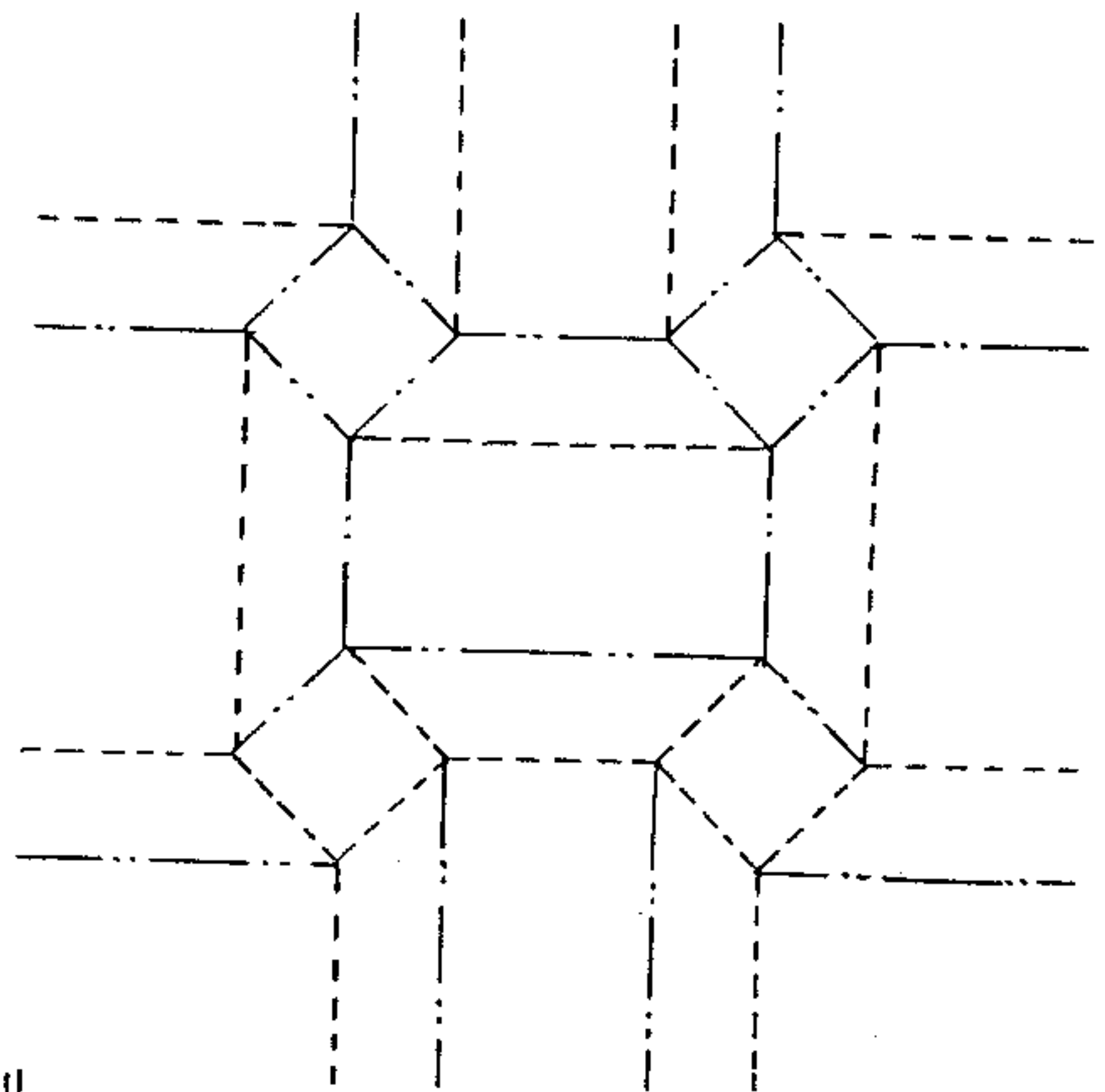


Fig. 2.4





(3)



(4)

(5)

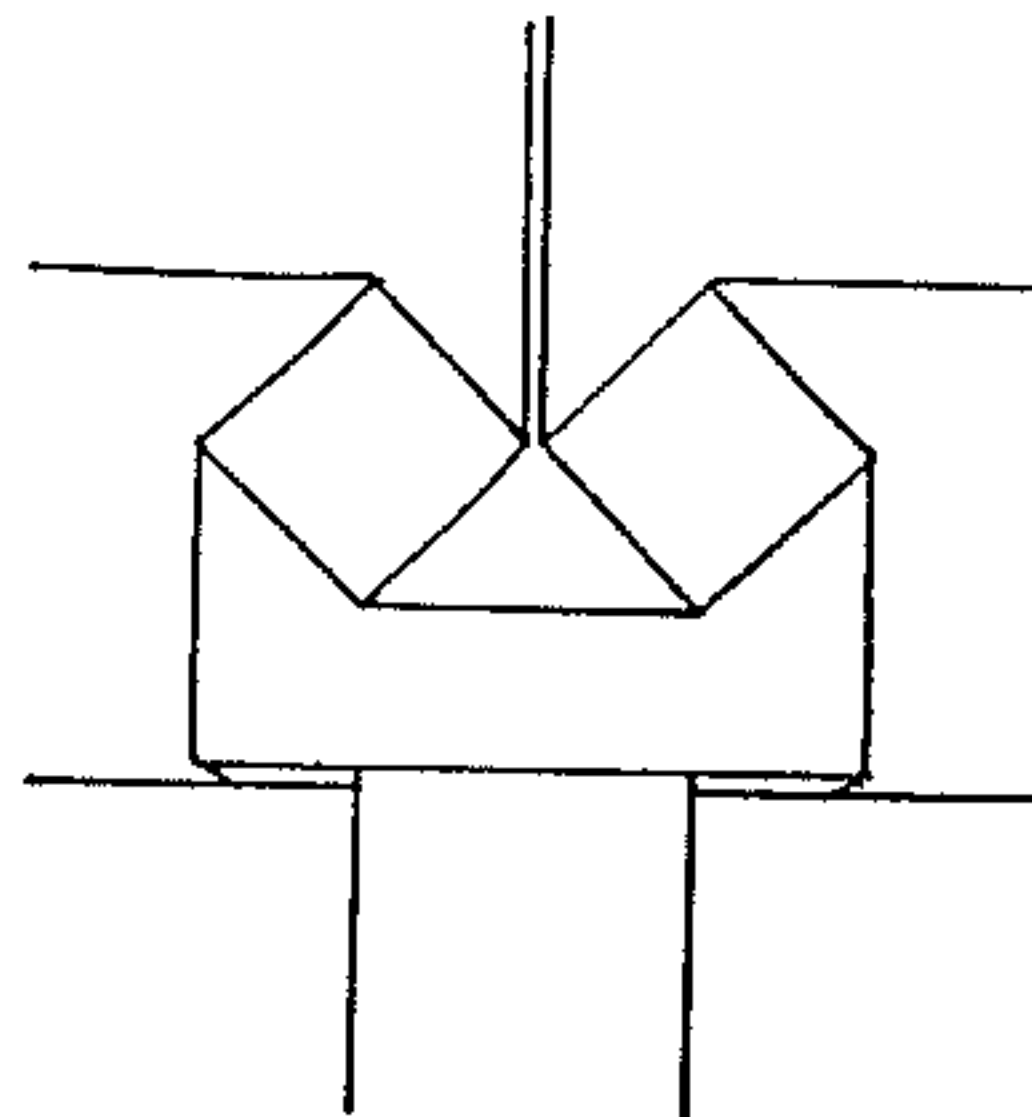


Fig. 2.5

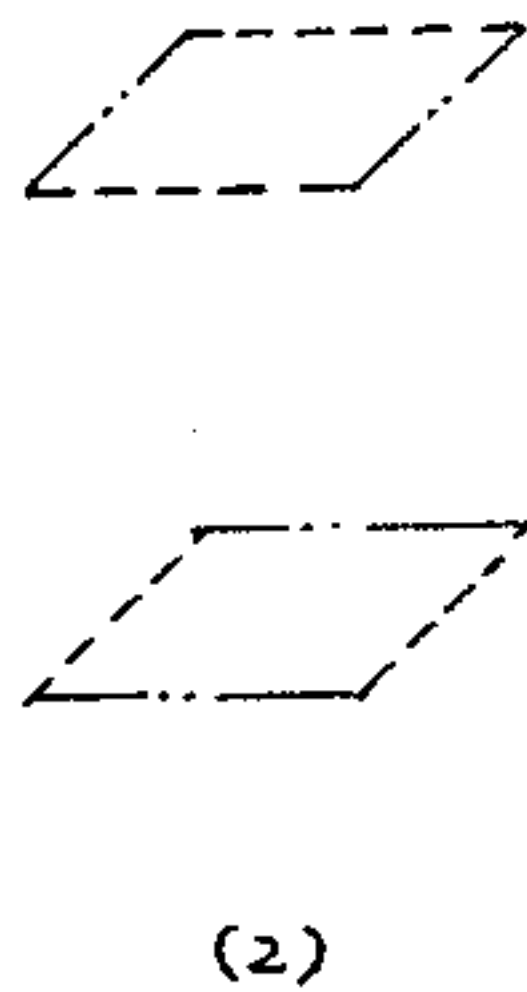
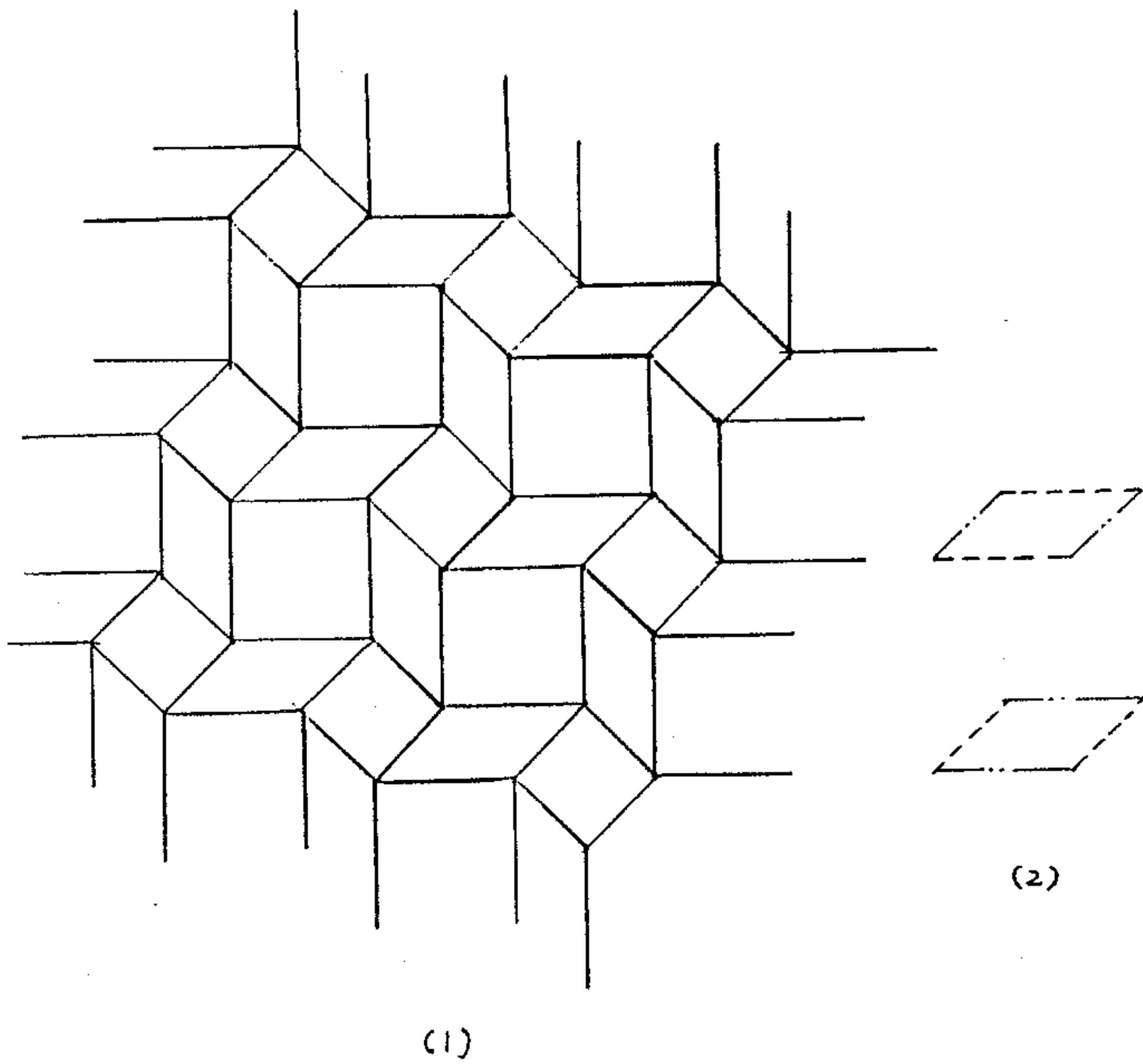


Fig 2.6

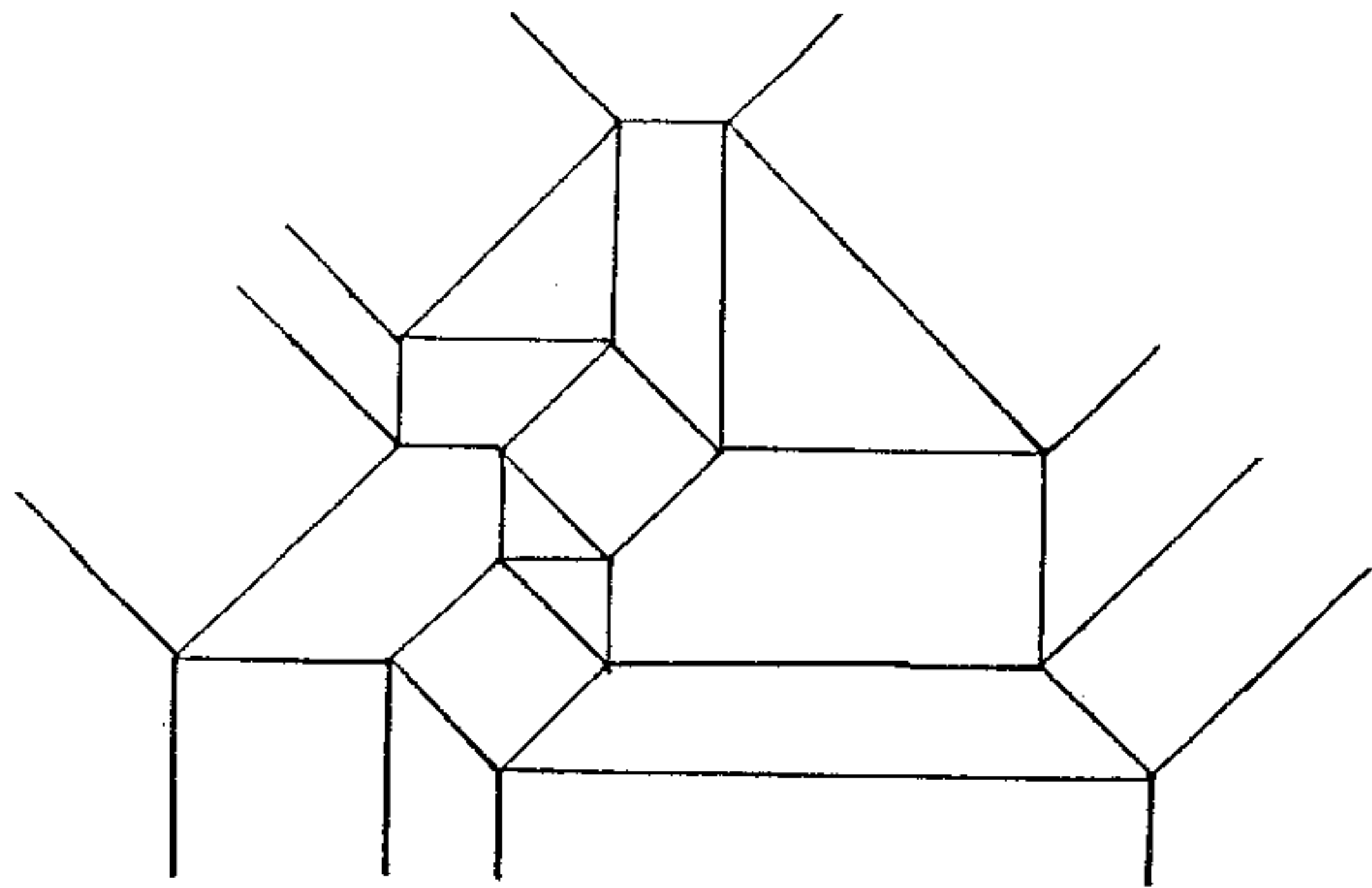


Fig 2.7

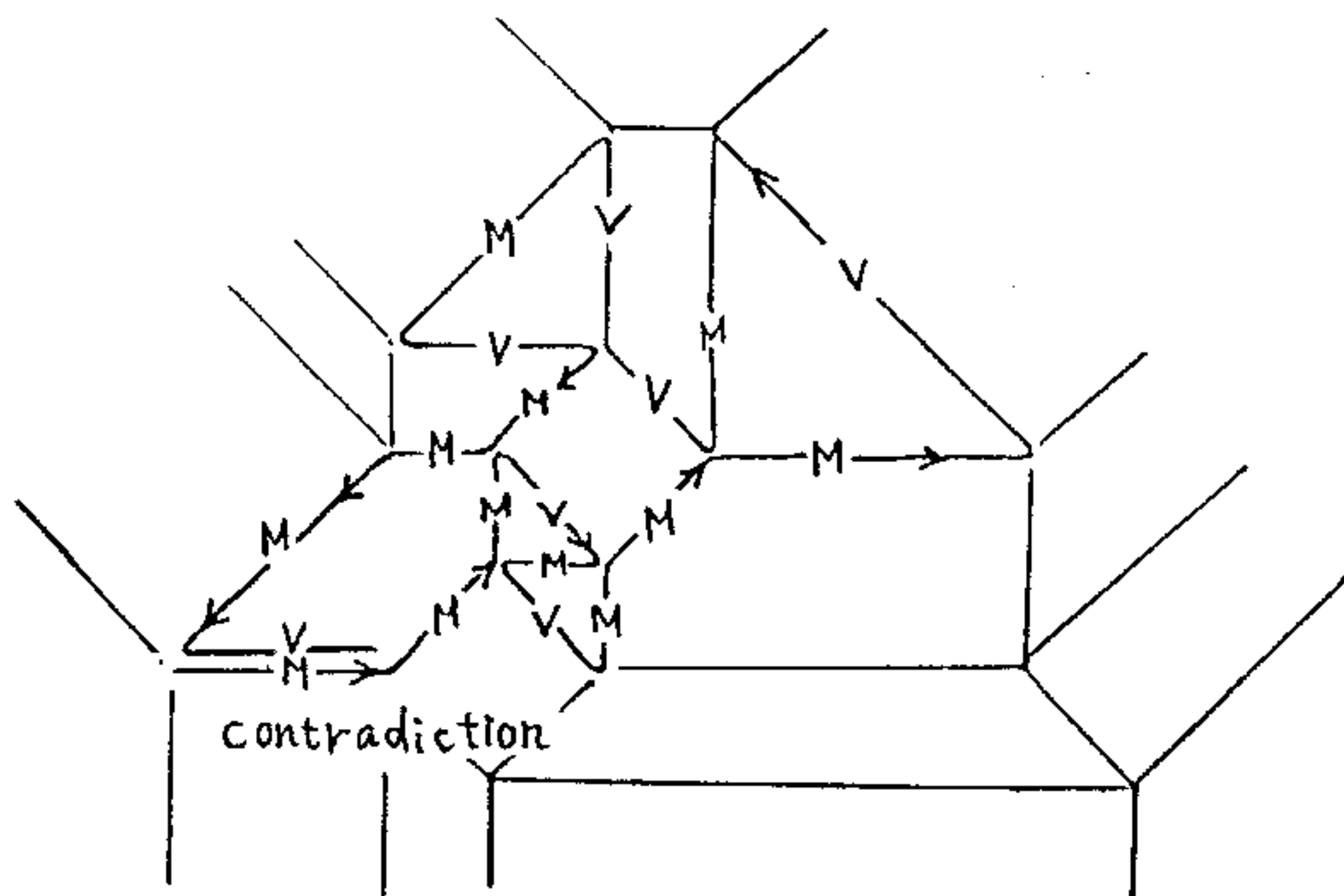
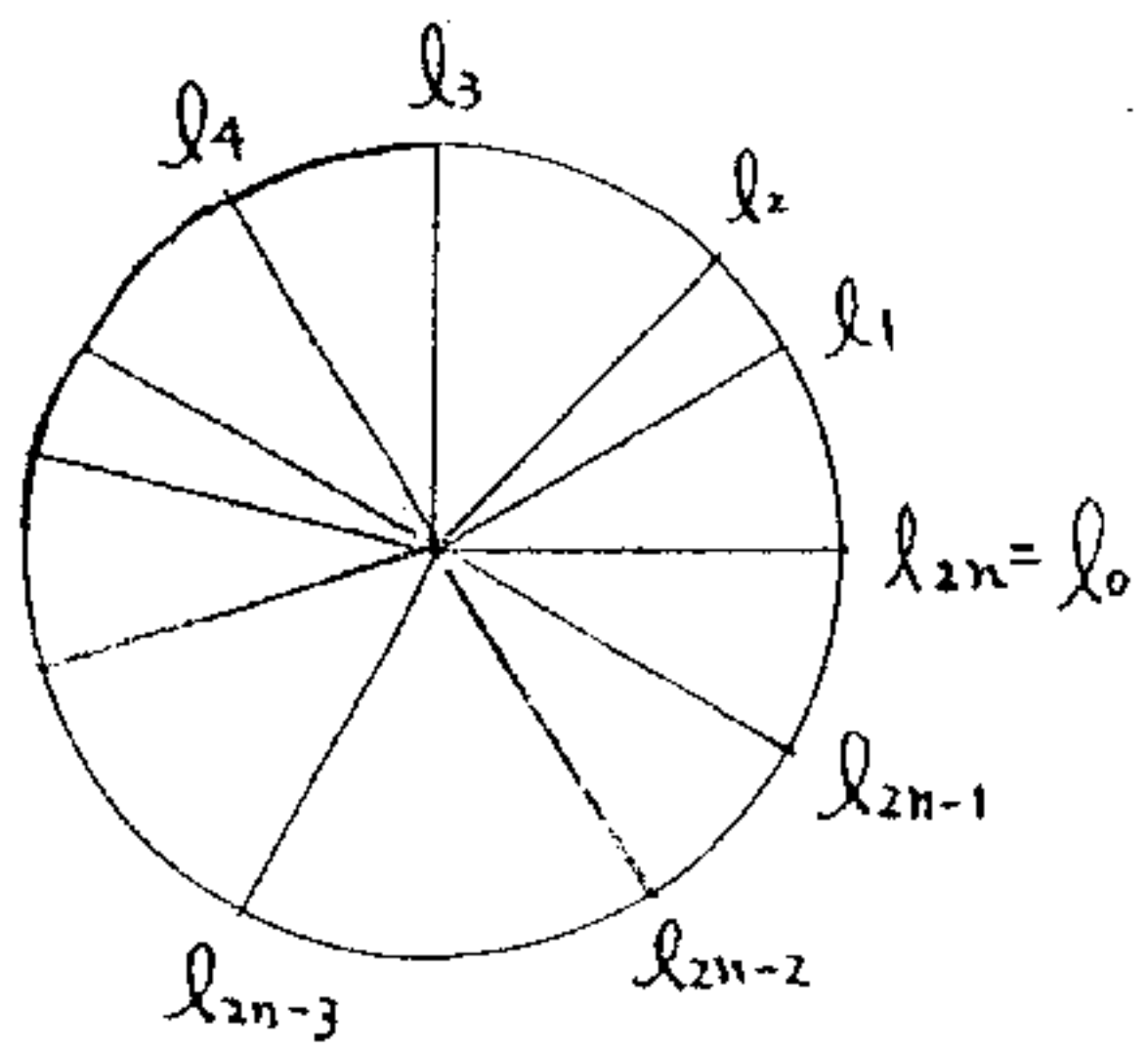
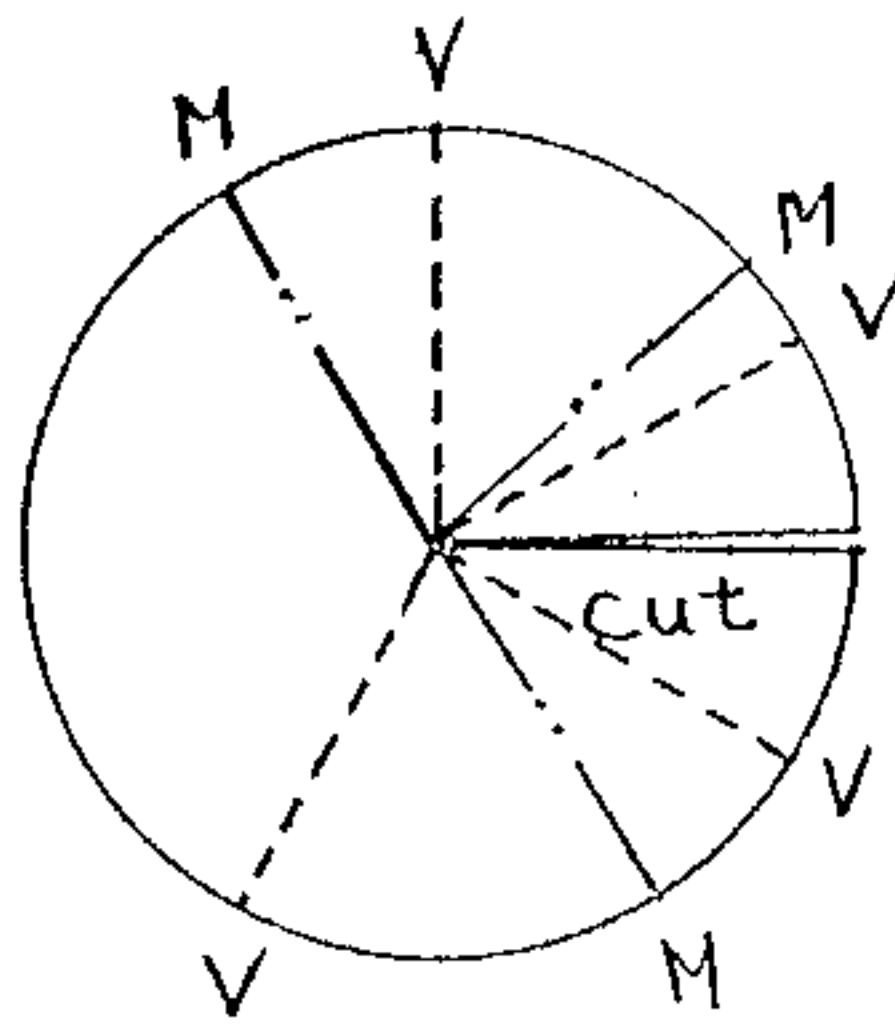


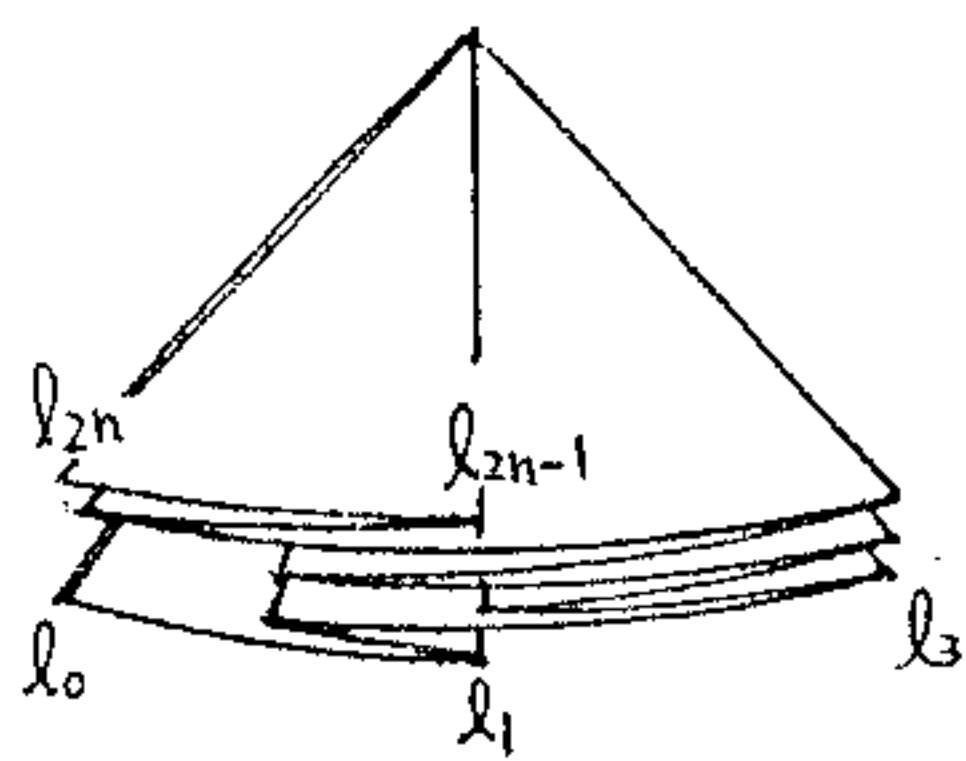
Fig.2.8



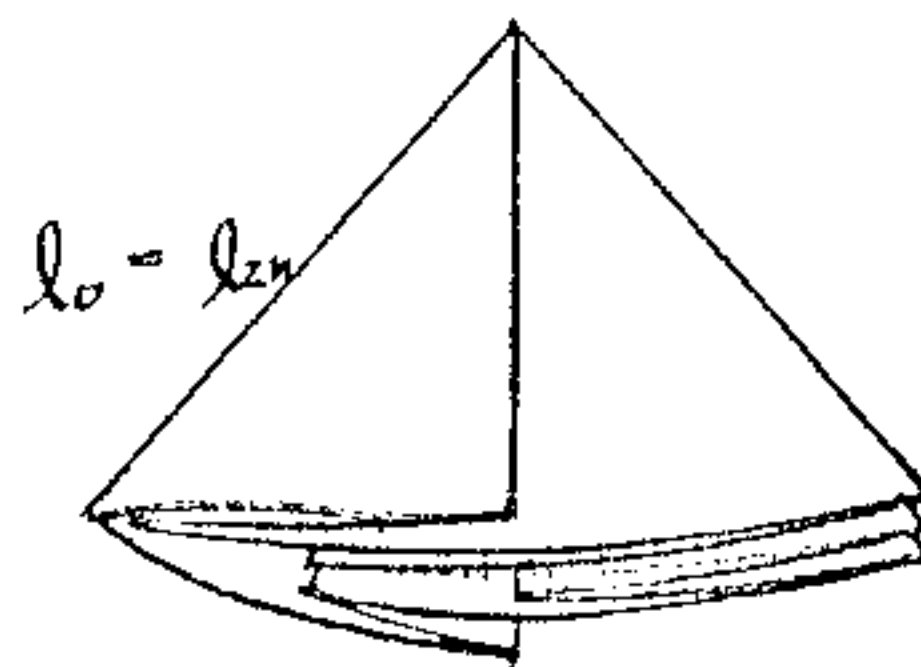
(1)



(2)



(3)



(4)

Fig. 2.9